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ABSTRACT

A discussion of autosegmental phonology (AP), a theory of phonological representation that uses graphs rather than strings as the central data structure, considers its principal constraint, the "No Crossing Constraint" (NCC). The NCC is the statement that in a well-formed autosegmental diagram, lines of association may not cross. After an introductory section, the syntax and semantics of autosegmental representations are considered, and a few important basic definitions and principles of graph theory are introduced. In section 3, two claims are examined, including: the claim that the NCC restricts the class of representations in planar and non-planar AP; and the belief of some proponents of three-dimensional AP that the necessity of non-planar representations has been demonstrated. Finally, some data exemplifying a number of interacting harmonies in Guyanese English that are amenable only to non-planar representation are presented. It is concluded that the NCC is invalid because it either incorrectly restricts the class of phonological graphs to planar graphs or else carries no force. (MSE)

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THE 'NO CROSSING CONSTRAINT' IN AUTOSEGMENTAL PHONOLOGY*

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1 Introduction

Autosegmental Phonology is a theory of phonological representation which employs graphs rather than strings as its central data structure (van der Hulst and Smith 1982). Phonological processes such as assimilation, harmony etc. are given derivational accounts similar to those employed in string-based approaches to phonology. But in line with the trend towards declarative formalisms in linguistic theory, Autosegmental representations and derivations are sanctioned not by explicitly ordered grammar rules, but by general 'principles' tempered by 'constraints', together with some language-specific rules. Well-formedness of an Autosegmental representation and its derivation are assessed by their adherence to and satisfaction of these 'principles', 'constraints' and rules. In this paper we shall consider the principal 'constraint' of Autosegmental Phonology, the so-called 'No Crossing Constraint' (Hammond 1988), and show that

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it does not, in fact, constrain the class of well-formed Autosegmental representations. (See section 2 for a definition of the N.C.C.)

We are not alone in examining the basis of the No Crossing Constraint. Sagey (1988), for instance, examines two interpretations of the diagrams used in Autosegmental Phonology, and shows that the No Crossing Constraint follows as a necessary consequence of one of these interpretations. She concludes that the ill-formedness of Autosegmental representations with crossing association lines derives from extralinguistic knowledge about two timing relations, 'precedence' and 'overlap'.

In this paper, we consider Autosegmental Phonology as a theory of grammar for a particular family of graphical languages (sets of graphs yielded by graph-grammar derivations). We are careful to distinguish the syntax of these languages (i.e. the *form* of phonological representations) from their semantics, that is, from possible *interpretations* of those representations. The fact that the No Crossing Constraint can be derived from a particular interpretation of Autosegmental representations suggests that Sagey's hypothesis involves phenomena that are not strictly speaking extralinguistic, but rather 'extrasyntactic', i.e. semantic in the terms just defined.

As part of our work in constructing computational implementations of nonlinear phonology in the field of speech synthesis, we have independently duplicated Sagey's result, and have also developed the stronger syntactic argument that the No Crossing Constraint (N.C.C.) is not a constraint at all, strictly speaking, since it does not restrict the class of well-formed phonological representations. The core of our argument can be briefly sketched as follows:

A distinction must be drawn between Autosegmental phonological representations, and diagrams of those Autosegmental phonological representations. Diagrams are not linguistic objects, but pictures of linguistic objects, and may have properties such as perspective, colour etc. which are of no relevance to linguistic theory. The N.C.C. is a constraint on diagrams, not on Autosegmental represen-

tations. When the conditions by which the N.C.C. restricts the class of diagrams are examined and linguistically irrelevant factors such as width or straightness of lines are removed, it is apparent that the *intention* of the N.C.C. is to enforce the following *planarity constraint*: Autosegmental representations are planar graphs. Thus, the planarity constraint is the defining distinction between the two varieties of Autosegmental Phonology, planar and nonplanar (i.e. multiplanar).

In planar Autosegmental Phonology the No Crossing Constraint has no place in *linguistic* theory, since it is *universally* the defining characteristic of planar graphs. In nonplanar Autosegmental Phonology the N.C.C. is unrestrictive, because *all* graphs can be drawn as 3-D diagrams with no lines crossing.

We consider our syntactic argument to be stronger than Sagey's semantic argument, since it is not dependent on a particular theory of the interpretation of phonological representations, but follows from general principles of graph theory alone.

The rest of this paper is set out as follows. In section 2 we consider the syntax and semantics of Autosegmental representations and we introduce a few important basic definitions and principles of graph theory. In section 3 we consider the veracity of the claim that the No Crossing Constraint restricts the class of representations in planar and nonplanar Autosegmental Phonology, and we examine the belief of some proponents of 3-D Autosegmental Phonology that the *necessity* of nonplanar representations has already been demonstrated. Finally, in section 4 we present some data exemplifying a number of interacting harmonies in Guyanese English that are amenable only to nonplanar representation.

2 Autosegmental Phonology and Graph Theory

We begin by considering the syntax and semantics of Autosegmental phonological representations, introducing definitions of the terminology which we employ in our subsequent argument. Initially, we must be careful to distinguish Autosegmental representations (A.P.R.s), which are linguistic objects, from both diagrams (which are pictorial objects) and graphs (which are mathematical objects).

Let us first consider the question 'What are A.P.R.s?' A naïve answer to this question is that they are diagrams i.e. pictures in journals, etc. This first hypothesis can easily be dismissed. Being pictorial objects, diagrams are necessarily flat. However, diagrams may have properties, such as perspective, that are not shared by phonological representations. For instance, it is possible to portray a two-dimensional object on a flat surface but with a three-dimensional perspective, e.g. by drawing a circle in 3-space as an ellipse in the plane of the paper. (We shall refer to perspectiveless diagrams as 2-D diagrams, and to diagrams with three-dimensional perspective as 3-D diagrams.) Therefore, diagrams in journals are not in themselves Autosegmental representations, but pictures of Autosegmental representations. What then, are Autosegmental representations?

A more sophisticated hypothesis, which does not fall prey to the immediate problems of the naïve hypothesis, is 'a phonological representation is a mathematical object that has precisely the 'important' diagrammatic properties that phonological diagrams have.' But this hypothesis begs the question as to which diagrammatic properties are 'important', and which are not. The resolution of this question is fundamental to this paper.

In Autosegmental Phonology, a phonological representation consists of a number of phonological objects (segments, autosegments and timing slots) and a two-place relation, called *association* (A), over those objects. In addition, the phonological objects in an Autosegmental representation are partitioned into a number of well-ordered sets, called *tiers*.

In Autosegmental *diagrams*[1], phonological objects are represented by alphabetic symbols, features or vectors of features, and the association relation by straight lines connecting each pair of objects that is in the association relation. Tiers are portrayed in Autosegmental diagrams by horisontal sequences of objects separated by spaces. The No Crossing Constraint is the statement that in a well-formed Autosegmental diagram, lines of association may not cross.

We shall not consider what A.P.R.s denote. A large number of views concerning this question have been advanced over the years, and it seems unlikely to us that agreement will ever be reached.

Despite the ongoing debate about the semantics of A.P.R.s, it is possible to demonstrate our claims concerning the nonrestrictiveness of the No Crossing Constraint from consideration of the *syntax* (i.e. form) of A.P.R.s alone. In order to do this, we first set out some elementary definitions and theorems of graph theory.

In mathematics, a collection of objects and a two-place relation defined over those objects (often with the explicit inclusion of endpoint maps π_1 and π_2, though these are usually omitted if multiple arcs are not permitted, cf. Rosen 1977) is called a *graph*.

Formally, a graph G is a tuple (V, E), (optionally with the addition of endpoint maps π_1, π_2) where V is any set of objects, called *vertices* in graph theory, and E a set of pairs of vertices, called *edges*.

The term 'vertex' is a general mathematical term for primitive objects in whatever domain is being modelled, and the term 'edge' is a mathematical term for each pair of objects in a relation. The *degree* of a vertex is the number of edges of which that vertex is a member.

The definition of a graph and the terminology of graph theory are completely independent of any particular drawing conventions that

[1]Figures (2) and (3) are typical Autosegmental diagrams.

may be used to represent a graph diagramatically. Graphs are abstract mathematical entities with no unique visible manifestation. In particular 'graph' is not synonymous with 'diagram', 'vertex' is not synonymous with 'point' and 'edge' is not synonymous with 'line'. We must thus be careful to distinguish graphs from their various possible diagrammatic instantiations. Vertices can be any type of object whatsoever, edges are simply pairs of vertices, and graphs are simply pairs of sets of vertices and edges.

A *bipartite* graph is one in which the set of vertices can be partitioned into two disjoint subsets N_1 and N_2, such that the set of edges is a subset of $N_1 \times N_2$. Since in Autosegmental Phonology the association relation holds only over objects on separate tiers (i.e. an object may not be associated with any other object on the same tier), the existence of tiers in Autosegmental Phonology ensures that its graphs are bipartite. For instance, N_1 might be the set of timing slots, and N_2 the set of melody segments.

Graphs in which the set of nodes can likewise be divided into n disjoint subsets are called n-partite graphs. An Autosegmental representation with an anchor tier and m melody tiers is thus *maximally* an $(m+1)$-partite graph, though it might, depending on what conditions are imposed on the association relation, be *minimally* merely a bipartite graph. This is the case if melody units can only be Associated to anchor units, and not to each other. Pulleyblank (1986: 14) advances this condition. It is also the case if the theory of tier organisation proposed by Clements (1985), in which each segment is a tree of melody-units, makes Autosegmental representations at most bipartite (since all trees are bipartite graphs). In multi-tier 2-D Autosegmental diagrams, association lines do not usually 'cross' tiers except at a node. The graphs denoted by these digrams are also (at most) bipartite. Since m-partite graphs where $m > 2$ are therefore prohibited in one way or another from occuring in the several varieties of Autosegmental Phonology, Autosegmental diagrams denote (amongst other things) bipartite graphs.

A complete bipartite graph of the form $K_{1,s}$ is called a *star*

graph. Such a graph has one node (the *root*) linked to each of the others (the *leaves*).

A *circuit* graph is a connected graph in which every node is of degree two. We define a *chain* to be a circuit graph with one arc removed. In a chain, every node is linked two two others except for two end-nodes, which are of degree one.

An *Autosegmental Phonological Representation (A.P.R.)* is a tripie $(G, L, \prec)$, where G is a graph (O, A, π_1, π_2) of the association relation, O is the set of phonological objects, L a partition of O, A a subset of $L^i \times L^j$ where L^i and L^j are members of L, $i \neq j$, π_1 and π_2 maps from A to O which pick out the endpoints of each association line, and $\prec$ a total order on each L^n.

In his 'Excursus On Formalism', Goldsmith (1976: 28) defines an Autosegmental phonological representation as a set of sequences L^i of objects a^i_j (each of which is a tier, which Goldsmith calls 'levels'), together with an ordered sequence A of pairs of objects whose first and second members are drawn from disjoint tiers. Apart from the total ordering of elements in the association relation A, the characterisation of Autosegmental phonological representations which we presented in the preceding paragraph is identical to Goldsmith's.

Our first line of attack on the N.C.C. is to show that it follows directly from Goldsmith's explicit total ordering of A.

Let $\trianglelefteq$ be the total ordering on A, let $\{(a, b), (c, d)\} \subset A$ and let $\{a, c\}$ and $\{b, d\}$ be in disjoint tiers. Goldsmith states (1976: 28) 'A in a sense organises the other levels' (i.e. the endpoints of A). Although it is not completely clear what he means by this statement, we claim that $(a, b) \trianglelefteq (c, d) \Leftrightarrow a \prec c$ and $b \prec d$. For if this is not so, the total ordering on A serves no purpose and should be dispensed with.

Theorem 1 *The N.C.C. follows from the total ordering of A.*

Proof 1 *If 'lines cross' (i.e. if* $a \prec c$ *but* $d \prec b$*) then neither* $(a,b) \trianglelefteq (c,d)$ *nor* $(c,d) \trianglelefteq (a,b)$*, in which case* $\trianglelefteq$ *is not total and A is not well-defined. A is only well-defined if lines do not cross, or in other words, if the N.C.C. holds.* □

But if the N.C.C. can be derived so trivially from mathematical properties of Autosegmental representations, it is not a *linguistic* constraint.

However, since the total ordering of association lines is an undefended stipulation of Goldsmith's, we shall proceed in the argument which follows in the belief that the N.C.C. is not vacuous, therefore abandoning the stipulation that A is totally ordered.

Since all Autosegmental phonological representations are graphs on which some further restrictions have been placed, all of the *universal* properties of graphs hold of Autosegmental phonological representations, together with some *special* properties. Autosegmental representations are a special kind of graph, but they are also subject to all the universal properties of graphs.

Our formal characterisation of A.P.R.s differs from Goldsmith's (1976: 88) attempt at formalisation only in the ordering of A. Our characterisation correctly captures all the necessary structural properties of Autosegmental representations, (division into tiers, well-ordering of tiers, adjacency and locality of neighbouring elements within a tier, the association relation), but Goldsmith's attempt to derive the N.C.C. from preservation of connected subsequences of segments under inversion of the association relation fails to work in a number of elementary cases, as he admits in two footnotes (Goldsmith 1976: 55, notes 5 and 6).

Having established that Autosegmental representations are graphs (a kind of formal object), we shall consider the relationship between the two kinds of Autosegmental diagrams (2-D and 3-D), and two kinds of graphs, planar graphs and Euclidean (nonplanar) graphs.

2.1 Planarity

A *Jordan curve* in the plane is a continuous curve which does not intersect itself. A graph G *can be embedded* in the plane if it is isomorphic to a graph drawn in the plane with points representing the vertices of G and Jordan curves representing edges in such a way that there are no crossings. A *crossing* is said to occur if either

1. the Jordan curves corresponding to two edges intersect at a point which corresponds to no vertex, or
2. the Jordan curve corresponding to an edge passes through a point which corresponds to a vertex which is not one of the two vertices which form that edge.

A *planar* graph is a graph which can be embedded in a plane surface.

A *Euclidean* graph is a graph which can be embedded in Euclidean space, that is, normal, three-dimensional space. All planar graphs are Euclidean, but not all Euclidean graphs are planar. That is, there are some Euclidean graphs which cannot be embedded in the plane.

The two kinds of graphs and diagrams we are considering are expressed in the following table:

Graphs:	Planar graphs	$\subset$	Euclidean graphs
Diagrams:	2-D diagrams	$\subset$	3-D diagrams

We have singled out the planar/Euclidean distinction for particular consideration, since it might be thought that there is a simple one-to-one relation between planar graphs and 2-D diagrams, and Euclidean graphs and 3-D diagrams. We shall demonstrate that this is not the case, and that this mistaken view underlies a number of problems with the N.C.C.

By definition, every planar graph can be drawn in the plane of the paper as a flat or perspectiveless network of points and noncrossing lines (a 2-D diagram); and every flat network of points and noncrossing lines represents a planar graph.

By definition, every 3-D network of points and noncrossing lines represents a Euclidean graph. We now show that the reverse case also holds.

Theorem 2 *Every graph can be embedded in 3-D space.*

Proof 2 *We shall give an explicit construction for the embedding. Firstly, place the vertices of the graph at distinct points along an axis. Secondly, choose distinct planes (or 'paddles') through this axis, one for each edge in the graph. (This can always be done since there are only finitely many edges.) Finally, embed the edges in the space as follows: for each edge joining two distinct vertices, draw a Jordan curve connecting those two vertices on its own 'paddle'. (We assume there are no edges joining a vertex to itself.) Since the planes or 'paddles' intersect only along the common axis along which all the vertices lie, none of the Jordan curves corresponding to the edges of the graph cross.* □

Theorem 3 *Every graph G can be drawn in a 3-D diagram as a network of points and (in perspective) noncrossing lines.*

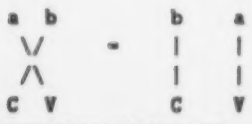
Proof 3 *Embed G in 3-D space. Draw G in perspective.* □

The fact that in Autosegmental representations, the set of vertices is partitioned into tiers, each of which is totally ordered, does not affect the validity of these Theorems. The single axis of vertices required for the construction used in the proof of Theorem 2 may be partitioned into subsets of well-ordered objects without affecting the result.

2.2 Paddle-wheel Autosegmental representations

Pulleyblank (1986: 12-14) considers limitations on the association relation. He argues that if the objects in every tier may only be associated with the objects ('slots') in a distinguished ('skeletal') tier, and not to the objects in any other tier, the theory which results is 'considerably more restrictive'. The graphs yielded by Pulleyblank's proposed restriction have come to be known as 'paddle-wheel' graphs (Archangeli 1985: 337) since they consist of a set of planar graphs which intersect along a shared tier, the skeleton (cf. 3 and 14 below). Since Autosegmental representations are graphs, Pulleyblank's claim must mean that versions of Autosegmental phonology which allow only 'paddle-wheel' graphs are 'considerably more restrictive' than theories which allow general graphs. Yet Theorem 2 shows that this is not correct. For Pulleyblank's claim to hold, there must also be restrictions on the composition of each tier other than linear ordering (in other words, restrictions on the *objects* in each tier), and on the *straightness* of association lines.

A number of Autosegmental phonologists (Clements and Keyser 1983: 11, Prince 1984: 235, Clements 1986, Pulleyblank 1986: 14) who subscribe to the 'paddle-wheel' theory claim that timing relations between Autosegments are dependent on the ordering of objects in the skeleton. In this case, the maximally parsimonious account is one in which autosegmental tiers are not explicitly ordered.[2] If only elements on the skeletal tier are ordered, then the N.C.C. has no force since



[2]Kaye (1985: 280,301-304) crucially requires nonskeletal tiers to be unordered, as does Lowenstamm and Kaye (1986), although this latter paper explicitly denies that phonology is three-dimensional, despite accepting the principles of Autosegmental phonology.

Figure 1: Two embeddings of a planar graph.

where C and V are on the skeletal tier and the tier $\{a, b\}$ is unordered.

The need for association lines to be straight for the N.C.C. to work can be demonstrated as follows. Consider the graph:

$$(\{t_1, t_2, x_1, x_2\}, \{(t_1, x_2), (t_2, x_1)\})$$

with partition into tiers $T_1 = \{t_1, t_2\}$, $T_2 = \{x_1, x_2\}$ and the order $t_1 \prec t_2$, $x_1 \prec x_2$ (1a). If the N.C.C. requires association lines to be straight, then this Autosegmental representation cannot be drawn without crossing lines (1a), and it would thus be *excluded* by the N.C.C. But if there is no such restriction on the straightness of association lines, this Autosegmental representation *can* be portrayed without crossing lines (1b), and thus the N.C.C. does not prohibit this A.P.R.

This demonstrates that the No Crossing Constraint is a condition on pictures, not phonological representations, since straightness of lines is a property of pictures, not linguistic representations. The straightness of association lines is conventional rather than for-

mal: it has never been explicitly defended in Autosegmental Phonology, it does not follow from other principles of the theory, and it is sometimes abandoned when it is convenient to do so (see for example McCarthy (1979/1982: 140), Archangeli (1985: 345), Prince (1987: 501), Pulleyblank (1988: 256,259), Hayes (1989: 300)). If the lines denoting the association relation need not be straight, then the N.C.C. will sometimes necessarily hold and at other times only contingently hold. The cases in which the N.C.C. contingently holds are those like (1), in which if the lines need not be straight, the N.C.C. can be circumvented. In such cases, the N.C.C. is nonrestrictive, and therefore cannot be linguistically relevant. However, in the cases in which the N.C.C. *necessarily* holds, it is indeed restrictive, for it limits the class of Autosegmental representations to planar graphs. In these cases adoption of the N.C.C. is equivalent to support for the hypothesis that Autosegmental representations are planar graphs.

Since the straightness of association lines is a property of Autosegmental diagrams, and not Autosegmental representations, Pulleyblank's position can only be maintained if there are constraints on tier composition which would diminish the force of our criticism. No such constraints have yet been established, although there are several possibilities:

1. Each tier bears a bundle of features, each of which cannot occur on any other tier.
2. Each tier bears a single phonological feature.
3. Each tier bears all of the segmental structure dominated by a single node of the Universal segment tree. (Clements 1985)
4. Each tier represents a morpheme. (McCarthy 1979/1982, Halle and Vergnaud 1980)

The first position cannot be maintained, since it is necessary in Autosegmental Phonology to allow more than one tier to bear *the same*

feature or features. Such proliferation of tiers has been employed in Autosegmental analyses of cases where a single feature (or set of features) has two different morphophonological functions. Prince (1987: 499) gives the following illustration of this:

> Arabic requires the same features to appear on different planes: for example, the affix /w/ is featurally identical to any other /w/, yet it clearly stands apart, tier-wise, because a root consonant may spread over it without line crossing in form XII

Halle and Vergnaud (1980) contains many examples just like this Arabic case.

Yet without the prohibition against the multiplication of features on different tiers, this position is simply the unconstrained null hypothesis that Autosegments are (unconstrained) bundles of features.

The second position (the 'single feature hypothesis', or S.F.H.) has been challenged on the grounds that it is empirically inadequate: it is sometimes desirable to treat two or more features as a single Autosegmental unit (when they have the same distribution, for instance). McCarthy's widely-supported analysis of Semitic morphology requires entire segmental melodies, not just single features, to be Autosegmental. The single feature hypothesis would not be sufficient to maintain Pulleyblank's claim concerning the restrictiveness of paddle-wheel A.P.R.s, unless multiplication of single features on several tiers (a move which is necessary to Autosegmental Phonology) were also prohibited.

The third hypothesis, proposed in Clements (1985), also falls foul of the need identified by McCarthy (1979/1982) and Prince (1987) for feature-structures to be replicated on several tiers. In every case, such replication undermines the restrictiveness of *any* proposal regarding tier composition, Clements, (1985) included.

The fourth hypothesis, the 'morphemic plane hypothesis', or M.P.H., is not sufficient to maintain Pulleyblank's claim, because it begs the question as to what phonological objects may constitute a morpheme. McCarthy (1989) shows that in the analysis of some languages (e.g. Mayan) it is necessary to represent vowel and consonant features on independent planes, although there is no evidence that vowels and consonants constitute separate morphemes.

We have thus shown that

- Autosegmental representations are graphs;
- 'paddle-wheel' graphs are no more restricted than general graphs;
- this fact is not diminished by the partition of objects into well-ordered tiers;
- no formal constraints on membership of phonological objects in tiers have yet been established.

From these, it follows that Pulleyblank's claim regarding the relative restrictiveness of 'paddle-wheel' A.P.R.s and Euclidean A.P.R.s is incorrect.

Let us conclude our consideration of the relationships between diagrams and graphs. We have shown that every planar graph can be drawn as a 3-D diagram. *However, not every 3-D diagram represents a planar graph. Some 3-D diagrams represent necessarily nonplanar graphs.* In order to understand the N.C.C., we are especially interested in the class of properly (i.e. necessarily) nonplanar graphs, which cannot be drawn in 2-D diagrams without crossing lines.

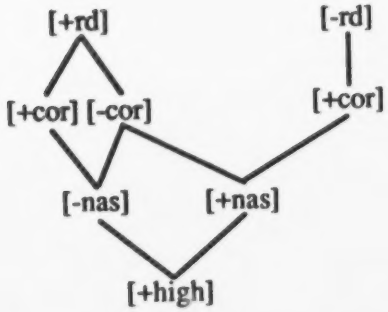


Figure 2: A planar graph.

3 Planarity and the N.C.C.

In the early days of Autosegmental Phonology (Goldsmith 1976), all Autosegmental diagrams were drawn as if to lie entirely in the plane of the paper. As we showed in the preceding section, however, if the No Crossing Constraint applies to Autosegmental representations, not diagrams, it defines a general, topological sense of planarity: namely, (planarity condition) a graph is topologically planar if and only if it can be embedded in a plane surface with (by definition of 'embedding') no edges crossing. Not all graphs can fulfil this requirement, however they are drawn, and it is therefore necessary to determine whether all Autosegmental representations can, if only in principle, be drawn in the plane. If some cannot, then A.P.R.s are in general nonplanar (whether they are portrayed as such or not), and the No Crossing Constraint is not restrictive.

If the N.C.C. applies to diagrams, it is a drawing convention, not a part of linguistic theory. But if it applies to Autosegmental representations, the planarity condition and the No Crossing Constraint are equivalent: the No Crossing Constraint has no specifically linguistic status, in that it is the defining characteristic of planarity, and should therefore be dropped from phonological analysis. We foresee that proponents of the N.C.C. might wish to argue that if the N.C.C. is retained, it is the planarity condition which is vacuous. But this argument is inadequate in two respects. Firstly, in addition to prohibiting Autosegmental representations which are necessarily nonplanar, the N.C.C. also excludes Autosegmental representations which may be portrayed in diagrams whose lines only contingently cross, even if there is some other way of drawing them in which no lines cross. Thus the N.C.C. prohibits some Autosegmental representations merely on the basis of the way in which they might sometimes be portrayed. The planarity condition, on the other hand, is a condition on Autosegmental representations, not diagrams of Autosegmental representations. It thus constrains linguistic (phonological) representations, not diagrams of linguistic representations. Secondly, to the extent that 'no crossing' is a universal property of planar graphs and not just those planar graphs employed in phonological theory, there is no reason for the mathematical definition of planarity to be imported into linguistics as a 'principle of Universal Grammar'. The relevant principle is 'Autosegmental representations are planar graphs' (if such is the case), not 'association lines must not cross'. For these reasons, given the equivalence of the N.C.C. and the planarity condition, those Autosegmental Phonologists seeking to defend the planarity hypothesis would do better to adopt the planarity condition directly (since it is a constraint on linguistic representations) than the N.C.C., which is a constraint only on *drawings* of linguistic representations.

Various phonologists (e.g. Archangeli 1985, Goldsmith 1985, Pulleyblank 1986) have found it convenient to generalise the planar representations of early Autosegmental phonology to 3-D, 'paddle-wheel', representations, in which several independent Autosegmental planes intersect along a distinguished tier of timing units. There is,

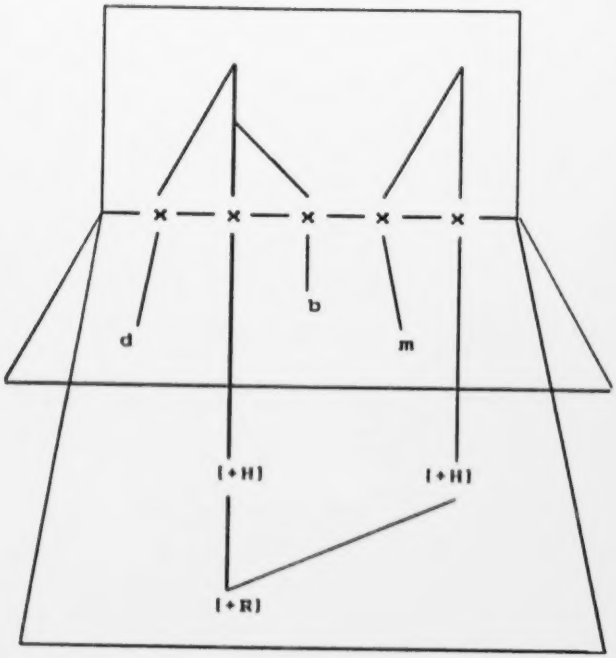


Figure 3: A paddle-wheel graph.

however, an important distinction between convenience and necessity. It is widely believed and commonly assumed by Autosegmental Phonologists that the *necessity* of 3-D representations has already been uncontentiously demonstrated. Archangeli (1985: 337), for instance, writes

> McCarthy's (1979;1981) analysis of Semitic *forced a truly three dimensional* phonological representation. [Our emphasis.]

Yet McCarthy (1979;1981) contain not a single diagram which even *appears* to be nonplanar, let alone a *necessarily* nonplanar representation.

Because the belief in the necessity for nonplanar Autosegmental representations is widespread, it has rarely been defended in the literature. As far as we are aware, no necessarily nonplanar phonological representation has yet been presented as a proof that Autosegmental representations are nonplanar.[3]

We shall attempt to defend our claim that the nonplanarity of Autosegmental representations has not yet been proven by establishing a necessary and sufficient criterion for a graph to be (necessarily) nonplanar. We shall then use this criterion to test the logic of the argument and examples adduced in support of the claim that phonological representations have already been shown to be necessarily nonplanar. We shall argue that the falsity of claims in the literature about 3-dimensionality arise from a failure to distinguish diagram conventions from genuine and uncontentious universal properties of graphs.

[3] In our examination of the literature, we have discovered one case of a diagram of an Autosegmental representation which is, in fact, demonstrably nonplanar. But this example is not offered in defence of the argument that Autosegmental representations are nonplanar. It is presented as part of a derivational account of timing in Ancient Greek. We discuss this case further below.

It is harder to show that a graph is *necessarily* nonplanar than that a diagram is 3-D. For a diagram to be 3-D it merely has to appear to be 3-D. A necessary and sufficient criterion for the nonplanarity of a graph G is:

Theorem 4 *(Kuratowski 1930)*[4] *G is nonplanar if and only if it contains a subgraph which is homeomorphic to either of the two graphs K_5 (the fully connected graph over five vertices) and $K_{3,3}$ (the fully connected bipartite graph over two sets of three vertices), shown in figure (4).*

(Two graphs are *homeomorphic* if they can both be obtained from the same graph by inserting new vertices of degree two into its edges.)

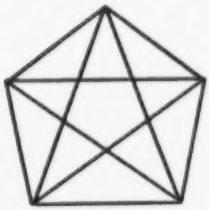
Note that Kuratowski's Theorem requires only that a subgraph of a graph is shown to be nonplanar in order to show that the whole graph is nonplanar.

Since the association relation is a bipartite graph, homeomorphism to $K_{3,3}$ is a necessary and sufficient condition for an Autosegmental representation to be necessarily nonplanar.

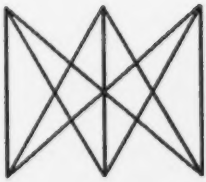
3.1 3-D Diagrams in the Autosegmental Literature

Figure (3), taken from Archangeli (1985), is typical of those diagrams of the association relation that are purported to be necessarily nonplanar. Archangeli's logic is inexplicit, but seems to be as follows (cf. Archangeli, 1985: 337): suppose there is a tier above the anchor tier (for instance, a consonant melody), and another tier below the anchor tier (for instance, a vowel melody). Then there are at least two 'paddles', one in the plane of the paper above the anchor tier,

[4]It is beyond the scope of this paper to present a proof of Kuratowski's theorem, since it is requires extensive and advanced familiarity with graph theory. A reasonably approachable presentation of the proof is Gibbons (1985: 77–80).



K5



K3, 3

Figure 4: Nonplanar graphs.

the other in the plane of the paper below the anchor tier. Now if yet another independent tier is called for (syllable templates, perhaps), yet another paddle, separate from the two in the plane of the paper, is required. Thus phonological representations with more than two melody tiers on separate paddles are necessarily nonplanar. This argument is erroneous. Figure (3) has three independent paddles, but it is not homeomorphic to either K_5 or $K_{3,3}$, and can be drawn in the plane with no lines crossing. Figure (5) is one possible plane embedding of (3). All the other examples of three-paddle Autosegmental diagrams that we are aware of from the literature (with the exception of the one we discuss below) also have plane embeddings. The graph of which figure (5) is a plane embedding is in no way affected by the manner in which it is portrayed. Since it is unchanged, it retains all the structure of figure (3), still supporting reference to all the relevant notions of locality (adjacency) and accessibility as in figure (3). Such an embedding is nothing other than a different way of looking at the same formal object.

The argument which Archangeli offers is, as far as we are aware, the only published attempt to establish the nonplanarity of Autosegmental representations. However, Archangeli's hypothesis has been generally accepted by Autosegmental phonologists, presumably because it is undeniably *convenient* to use 3-D diagrams in Autosegmental Phonology. We will examine some more examples of 3-D diagrams taken from the literature and show why, like Archangeli's example, they are not *necessarily* nonplanar.

Pulleyblank (1988). Like Halle and Vergnaud (1980), Pulleyblank (1988) subscribes to the M.P.H., stating:

> Following McCarthy (1981;1986) among others, I assume that the melodic content of different morphemes enters the phonology on distinct tiers (planes).

But Pulleyblank's examples contain at most only two morphemes, and thus all of them represent graphs which may be embedded in the

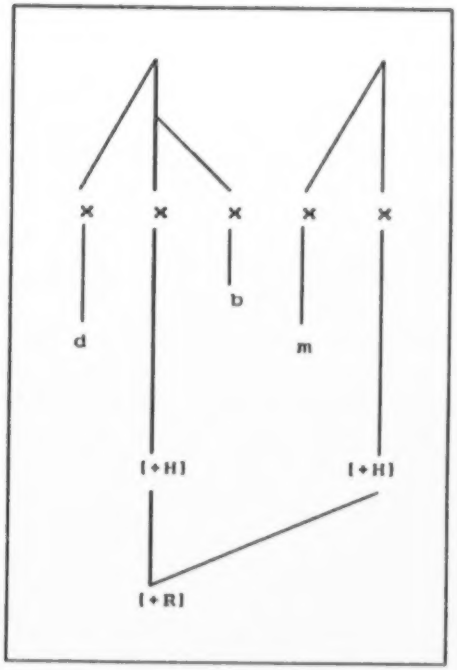


Figure 5: A planar embedding of a paddlewheel graph.

plane. None of Pulleyblank's 'apparently' 3-D diagrams (44), (50) represent *necessarily* nonplanar graphs.

Halle and Vergnaud (1987). Halle and Vergnaud argue that Autosegmental representations consist of several intersecting planes, and that they are therefore 'three dimensional' (by which, since they are talking about linguistic representations, not diagrams, we assume they must mean nonplanar). The rhetorical structure of their argument makes liberal use of conjunctions such as *thus, since, in fact* and *therefore* to build the appearance of a logically coherent, incremental argument, but this impression is deceptive because their argument is defective in several respects. Because it is one of the few papers in which an argument for the 3-D nature of Autosegmental representations is explicitly presented, we shall go through it very carefully, emphasising the unsupported conclusions.

> Autosegmental phonology has made it clear that tones must be represented as a sequence of units (segments) that is separate and distinct from the sequence of phonemes — in other words, that in tone languages phonological representations must consist of two *parallel* lines of entities: the phonemes and the tones. (Halle and Vergnaud 1987: 45).

The conclusion that the sequence of phonemes and the sequence of tones are *parallel* is unsupported. It is true that in Autosegmental diagrams, tiers always are parallel, but no Autosegmental phonologist has ever even attempted to demonstrate that 'phonological representations *must* consist of two parallel lines'.

> Since two parallel lines define a plane, we shall speak of the *tone plane* when talking about representations such as those in (1). (Halle and Vergnaud 1987: 45).

Two parallel lines do indeed define a plane, but Halle and Vergnaud have not established that associated tiers are parallel.

The next step in Halle and Vergnaud's argument is to show that stress, like tone, is autosegmental.

> We propose to treat stress by means of the same basic formalism as tone — that is, by setting up a special autosegmental plane on which stress will be represented and which we shall call the *stress plane*. (Halle and Vergnaud 1987: 46).

> It is not an accident that the bottom line both in the tone plane and in the stress plane is constituted by the string of phonemes representing the words. In fact, all autosegmental planes intersect in a single line, which as a first approximation may be viewed as containing the phoneme strings of the words. Autosegmental representations are *therefore* three-dimensional objects of a very special type: they consist of a number of autosegmental planes (to be geometrically precise, half-planes) that intersect in a single line, the line of phonemes. (Halle and Vergnaud 1987: 46).

This displays the same false reasoning as Archangeli (1985), discussed above. The establishment of several independent tiers linked to a common core is not sufficient to *prove* that A.P.R.s are *necessarily* nonplanar. It *is* sufficient to motivate the use of 3-D diagrams for clarity of presentation, but expository convenience is not a relevant factor in assessing the nature of phonological representations.

> We have argued that stress is represented on a separate plane from the rest of the phonological structure. It has been proposed elsewhere that other properties of morphemes, such as tone (Goldsmith 1976) and syllable

> structure (Halle 1985), are also to be represented on separate planes. Therefore, a morpheme will in general be represented by a family of planes intersecting in a central line. Given this formalisation, the combination of morphemes into words will involve a combination of families of planes. (Halle and Vergnaud 1987: 54).

Even if we grant that the tiers in an A.P.R. are parallel, and therefore do indeed define a family of planes, it does not follow that such a family of planes defines a three-dimensional object. While it is undoubtedly conceptually and pictorially *convenient* to picture a family of planes as forming a three-dimensional object, it is geometrically quite possible for a family of planes to lie in the same planar space.

Halle and Vergnaud even fabricate supporting evidence for their conception of phonological structure. They claim that:

> McCarthy (1986) has proposed that the separate autosegmental planes of Semitic morphology are the result of the fact that distinct morphemes must be represented on separate planes — for example, as in (20). (Halle and Vergnaud 1987: 54)

But unlike Halle and Vergnaud (1987) and Goldsmith (1985), McCarthy's (1986) article contains no autosegmental representations that are even *apparently* nonplanar (and no 3-D diagrams), let alone *necessarily* nonplanar. The 3-D diagram which Halle and Vergnaud credit to McCarthy is their own, not McCarthy's.

The structure of Halle and Vergnaud's argument can be summarised as follows:

1. Autosegmental tiers are parallel to the skeleton.
2. Therefore, each tier defines a (half-)plane.

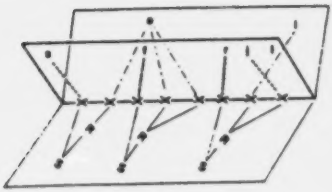


Figure 6: A 3-D diagram.

3. An autosegmental representation may contain several autosegmental tiers.

4. Therefore, an autosegmental representation consists of a family of intersecting (half-)planes.

5. Therefore, an autosegmental representation is a three-dimensional object.

Their argument does not go through, however, since the first proposition is unsupported and the final conclusion does not follow from the premisses.

Halle (1985). Although 3-D diagrams are rare, even in the writings of such proponents of '3-D Phonology' as Halle and Vergnaud, Halle (1985) presents a 3-D diagram, a representation of the Arabic word *safaarij* 'quinces' (6). However, no subgraph of the graph represented in this diagram is homeomorphic to $K_{3,3}$ or K_5, and thus (6) portrays a planar graph.

Halle (1985) is clear that diagrams such as (6) are not to be confused with the A.P.R.s that they denote. He states:

> information about the phonic shape of the words is stored in a fluent speaker's memory in the form of *a three--dimensional object that for concreteness one might picture* as a spiral-bound notebook. (Halle 1985: 101, our emphasis).

> I have tried to present *a picture* of this type of representation in Figure [6]. (Halle 1985: 112, our emphasis).

Moreover, Halle is clear that the diagrams of '3-D Phonology' are a *notation* for Autosegmental Phonological Representations, rather than being the representations themselves:

> there are no promising alternative *notations* to the multi-tiered autosegmental representation that has been described here. (Halle 1985: 112, our emphasis).

Yet, as we have argued throughout this paper, arguments for the felicity or utility of 3-D diagrams, or in other words, pictures of A.P.R.s do not constitute evidence the necessary nonplanarity of those representations.

Clements (1985). Clements (1985) examines two models of segmental organisation.

> To clarify our ideas, it would be useful to contrast two possible models of multi-tiered feature representation, representing opposed views of hierarchical organisation. (Clements 1985: 227)

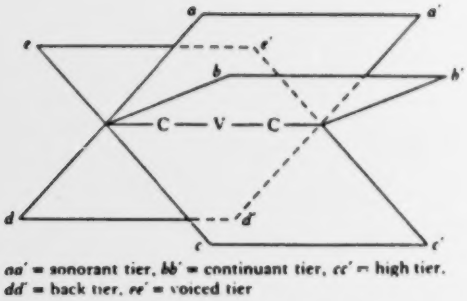


Figure 7: 3-D diagram with skeletal core.

In the first model, a segment is a star-graph whose root node is a skeletal object, whose leaf nodes are autosegments, and whose edges are association lines. The sequence of leaf nodes in adjacent segments forms tiers, and the sequence of root nodes forms a skeletal tier (7). Phonological representations are thus:

> multi-tiered structures in which all features are assigned to their own tiers, and are linked to a common core or 'skeleton'. (Clements 1985: 227)

Clements uses the metaphor of an open book[5] to describe such graphs:

[5]Halle is fond of the ring-bound notebook metaphor.

> In such a conception, a phonological representation resembles an open book, suspended horisontally from its ends and spread open so that its pages flop freely around its spine. The outer edge of each page defines a *tier*; the page itself defines a *plane*, and the spine corresponds to the *skeleton*. (Clements 1985: 228)

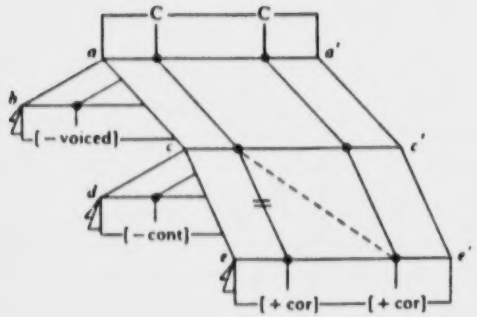
Each segment in this model is a star-graph consisting of a skeletal slot linked to the features which constitute that segment, each on its particular tier. The linear extension of a star-graph is a 'paddle-wheel' graph.

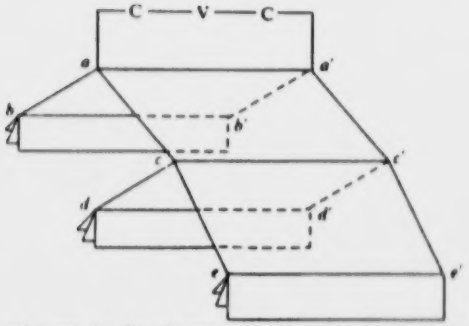
Clements contrasts this view with an alternative model in which each segment is not a star-graph but a tree-graph (8).

> This conception resembles a construction of cut and glued paper, such that each fold is a class tier, the lower edges are feature tiers, and the upper edge is the CV tier. (Clements 1985: 229)

Like other writers, Clements provides a number of appealing arguments for using 3-dimensional diagrams, and indeed offers empirical evidence in support of his position. But nowhere does he demonstrate that the evidence he musters explicitly proves that autosegmental representations are necessarily nonplanar graphs. All that he demonstrates is that it is convenient for expository reasons, simplicity etc. for A.P.R.s to be multiplanar objects of a particular type.

Both of the models which Clements compares are capable of supporting the Single Feature Hypothesis of tier content, but the second, tree-structured model is not capable of supporting the Morphemic Plane Hypothesis, as it is a highly specific theory of tier content. To the extent that morphemes may be phonologically arbitrary, in the manner described by Prince (1987: 449) and discussed above, it is inadequate as a constrained theory of noncatenative morphology.





aa' = root tier, *bb'* = laryngeal tier, *cc'* = supralaryngeal tier,
dd' = manner tier, *ee'* = place tier

Figure 8: 3-D diagrams with tree-structured segments.

There are, of course, many theories of segmental organisation consistent with all the principles of Autosegmental Phonology, other than the two which Clements singles out for consideration. Goldsmith (1976: 159), in which segments are *chains* of Autosegments, or Pulleyblank (1986: 13),[6] in which noncore tiers may be associated to other noncore tiers, are two attested alternatives, and others are possible.[7]

Goldsmith (1985). In this paper, Goldsmith employs 3-D diagrams for the first time. (It is not clear whether Goldsmith, Halle or Clements was first to do this. In each cases, the earliest publication of 3-D diagrams was in 1985.) However, Goldsmith (1985) uses 3-D diagrams for expository purposes only, and makes no theoretical claims about them. He states:

> The seven vowels of Mongolian are the seven vowels that can be created by the combinations of the feature [*front*] (represented as [i]), the feature [*round*] ([u]), and the feature [*low*] ([a]). These combinations arise through the association of a skeletal position with segments on three distinct tiers, on for each of these three features. This is illustrated in [*fig. 9*], where I have attempted to use perspective to represent four distinct tiers. (Goldsmith 1985: 257)

In his conclusion, he states:

> if the spirit of the analyses of Khalkha Mongolian, Yaka, Finnish, and Hungarian that are presented here is fundamentally correct, then the revisions of our conception

[6] Pulleyblank discusses, but does not subscribe to this view.

[7] For instance, segmental structure might quite plausibly represented by directed acyclic graphs, as in Unification Phonology, *circuits* or *wheels* (star-graphs with leaf-to-leaf association cf. Wilson, 1985: 19).

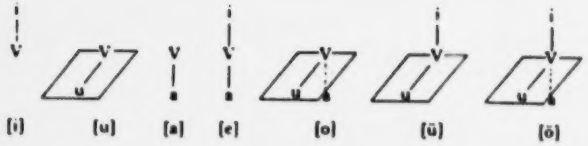


Figure 9: The vowels of Khalkha Mongolian.

> of phonological representation that we must adapt to are far-reaching, affecting both our view of autosegmental geometry and our understanding of traditionally segmental features. We will have to come to grips with truly rampant autosegmentalism (Goldsmith 1985: 271)

But unlike Halle and Clements, Goldsmith does not claim that A.P.R.s are three-dimensional objects.

Pulleyblank (1986). There are no necessarily nonplanar graphs in Pulleyblank (1986), although he does present a few considerations on the topology of Autosegmental representations. Like Halle and Vergnaud, Pulleyblank takes the view that:

> Nasality may be represented on a separate tier, vowel harmony features may be autosegmentalized, etc. This means that a language may require several independent (but parallel) tiers in its phonological representation. (Pulleyblank 1986: 12)

Just like Halle and Vergnaud, Pulleyblank slips in the unsupported assertion that if several independent tiers are required, they must be parallel, an assumption which is crucial to the hypothesis that tiers are organised into planes.

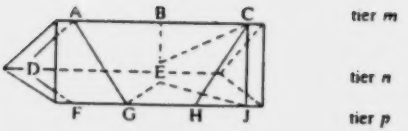
Pulleyblank considers two types of nonplanar Autosegmental representations. The first possibility which he considers is that each tier may be associated to any other. The only *formal* argument which Pulleyblank gives for rejecting this view is that tier-to-tier association can lead to contradictions in the temporal sequencing of autosegments. Commenting on figure (10), his example (21), he says:

> In (21), segments A and C have the value E on tier *p*; segment B, on the other hand, has the value F by virtue of the transitive linking B — D — F. But note that F *precedes* E in (21a), while it *follows* E in (21b)! In other words, the representation in (21) has as a consequence that the temporally ordered sequence EF is nondistinct from the sequence FE. (Pulleyblank 1986: 13)

This argument is flawed because association is not a transitive relation. If association *were* transitive, then the temporal interpretation of contour segments and geminates would be logically paradoxical (Sagey 1988: 110).

Furthermore, if temporal sequence is determined by the order of core elements, nonskeletal sequence is redundant. If nonskeletal tiers are unordered, then the apparent problem which Pulleyblank identifies vanishes.

Pulleyblank proposes an alternative type of nonplanar representation, paddle-wheel graphs, by adopting the restriction that Autosegmental tiers can only link to slots in the skeletal tier. He claims that the effect of this constraint is a 'considerably more restrictive



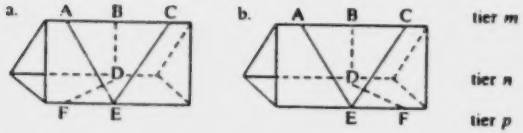


Figure 10: Direct tier-to-tier linking.

multi-tiered theory', a claim which we challenged above.[8] The only example of a 3-D diagram of a paddle-wheel graph which Pulleyblank presents includes no association lines at all, and it is therefore (trivially) planar.

McCarthy (1981). McCarthy (1981) includes none of the apparently 3-D diagrams of his earlier thesis (McCarthy 1979/1982), although the material in this paper is an abridged version of parts of that work. The framework is that of *n*-tiered autosegmental phonology without organisation into planes à la Halle and Vergnaud. In fact, quite contrary to Halle and Vergnaud, McCarthy has diagrams such as (11) (McCarthy 1981: 409 fig. 53) in which the CV 'core' occurs twice, in order to show the morphological correspondence between the first binyam and pɔ?al?al forms. (This is not a phonological representation, but a declarative formulation of reduplication.)

Halle and Vergnaud (1980). Although Halle and Vergnaud do not present any 3–D Autosegmental diagrams, they argue that 'the phonological representation is a three dimensional object' (Halle and Vergnaud 1980: 101) in the following manner.

> Its core is constituted by a linear sequence of slots — the skeleton. Each morpheme of the word is represented by a sequence of distinctive feature complexes ...the MELODY. (Halle and Vergnaud 1980: 101)

They accept the proposals of Autosegmental phonologists concerning the conditions which govern the linking of melody tiers to the skeleton, and claim that:

[8]The theory which results is considerably more *restricted*, but that is a different matter. Generative grammars of a particular type are certainly made considerably more restricted if their nonterminal symbols must all be words over the Cyrillic alphabet, but no more restrictive for all that.

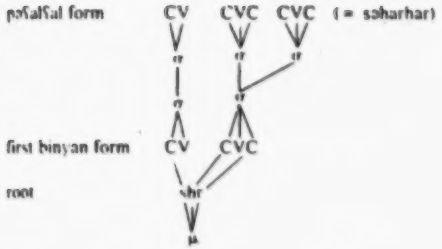


Figure 11: A diagram with two CV 'cores'.

> The lines that link the melody with skeleton define a plane. Thus, the phonological representation of a word contains as many planes as there are morphemes in the word. (Halle and Vergnaud 1980: 101)

This argument suffers from the same logical fallacy as that of Archangeli (1985): the (undisputed) clarity of presentation afforded by drawing subsets of the association relation over each individual morpheme's melody and the skeleton does not amount to a *proof* that planar Autosegmental representations are formally inadequate. Furthermore, as we argued above, without restrictions on what phonological material can constitute a morpheme, the departure from planar representations which Halle and Vergnaud support diminishes the force of the N.C.C. to the extent that it ceases to be restrictive.

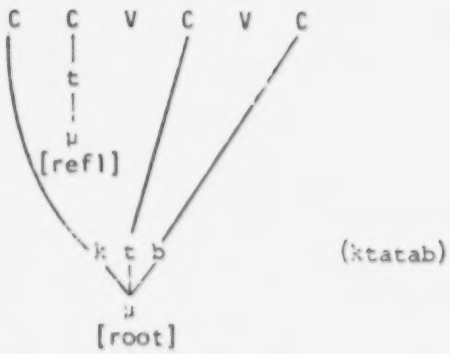
McCarthy (1979/1982). McCarthy's (1979/1982) thesis extended Goldsmith's (1976) Autosegmental theory of tonal phenomena to the nonconcatenative morphology of Semitic languages. There are no diagrams in this thesis which are even apparently 3-D, and nowhere in the text is the possibility of multiplanar (as opposed to multi-tiered) representations raised, although six of McCarthy's examples might, with generosity, be taken as attempting to portray Autosegmental graphs using perspective. These are reproduced in (12). Even if these examples are taken to be 3-D diagrams, they do not portray nonplanar graphs. Since they are all drawn on a plane surface with no crossing lines, they all portray planar graphs.

Goldsmith (1976). Goldsmith (1976) concentrates on two-tier Autosegmental representations, those with just a phoneme tier and a tone tier. He considers extending this formalism to multi-tiered Autosegmental representations (of which he presents a planar example portrayed in a 2-D diagram), but does not raise the possibility of 3-D diagrams or nonplanar Autosegmental representations.

We have shown that the logic which underlies the common belief that a necessarily nonplanar Autosegmental representation already exists is mistaken. This is surprising, for under the conventional assumption concerning the universality and homogeneity of language, demonstration that just one Autosegmental graph is necessarily nonplanar is necessary and sufficient for rejection of planar Autosegmental Phonology and 2-D diagrams, as inherently too restrictive.[9]

One such graph has in fact been portrayed in the Autosegmental literature (there are perhaps others too), in Wetsels (1986). In

[9]This is exactly parallel to a case from the history of Context-Free Phrase Structure Grammars. Throughout the 1960s and 1970s it was believed and taught by grammarians that Chomsky (1963: 378–379) and others had proved that English was not a Context-Free Language. In the early 1980s, however, these 'proofs' were shown to be fallacious in various respects (Pullum and Gazdar 1980), and it was not until some years later that respectable proofs of this widely-believed fact were actually constructed (Manaster-Ramer 1983, Huybregts 1984, Shieber 1985, Culy 1985).



a. XII b. XIII c. XIV d. XV

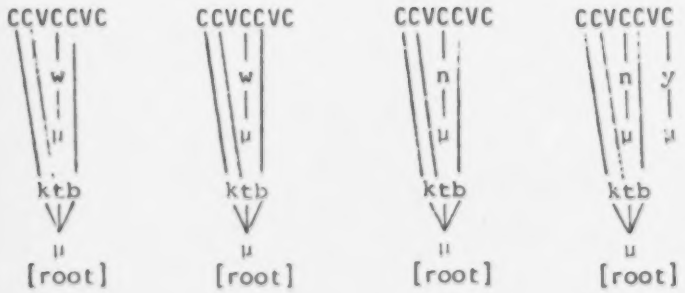


Figure 12: Autosegmental representations of Arabic.

the course of an Autosegmental derivation Wetzels gives a few diagrams of Autosegmental representations which contain $K_{3,3}$ as a subgraph. However, Wetzels's example is not presented as a proof that Autosegmental representations are nonplanar. Since he does not remark on the fact that his examples are nonplanar graphs, he appears to believe that the nonplanarity of Autosegmental representations has already been established. Furthermore, since Wetzels's example is from Classical Greek, and is therefore not amenable to first-hand verification, and since his analysis may be called into question, as a demonstration of nonplanarity it is not as uncontentious as is desired for a result to be established. We shall therefore present a synchronic example of an Autosegmental representation which is *necessarily* nonplanar. We shall establish the necessity of 3-D diagrams in Autosegmental Phonology by presenting an Autosegmental representation which is homeomorphic to $K_{3,3}$.[10]

4 A Necessarily 3-D Diagram

Consider a phonological representation with three anchor units on one tier, three autosegments on one or more other tiers, and a line of association between each anchor and each autosegment. Such a graph cannot be drawn without crossings on a plane surface, since it is homeomorphic to $K_{3,3}$.

There is no linguistic reason why such a representation might not be motivated in certain cases. Wetzels's example (13) is one such case. Two more are illustrated in (14), which shows the distribution of backness, rounding and nasality over three timing units in the pronunciation of the words 'room' and 'loom' by a Guyanese English speaker. Both of the graphs portrayed in (14) are homeomorphic to $K_{3,3}$, and thus they are necessarily nonplanar. In order to demonstrate that the graphs portrayed in (14) are the correct

[10] Whether we are successful or not in advancing this example, however, does not affect our general argument concerning the vacuity or nonrestrictiveness, as the case may be, of the N.C.C.

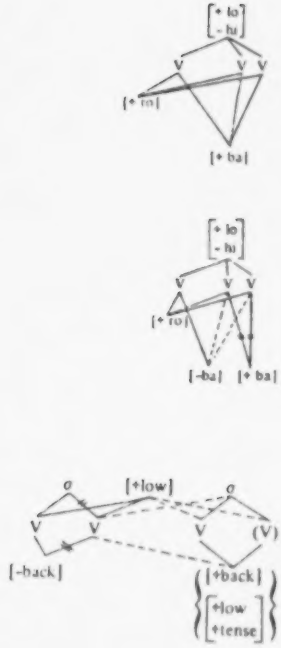


Figure 13: Nonplanar Autosegmental representations.

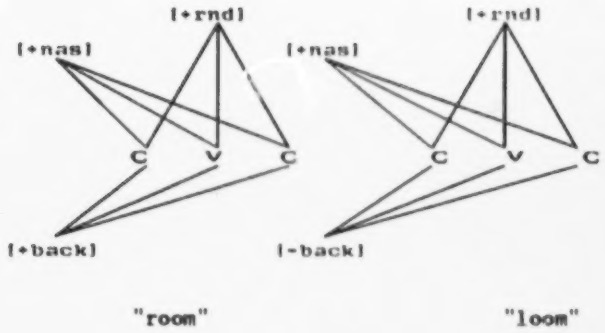


Figure 14: Necessarily nonplanar Autosegmental representations.

Autosegmental representations of the two words, we must establish that the features [*back*], [*round*], and [*nasal*] are indeed independent autosegmental features. We shall demonstrate that this is so by showing that they are lexically associated with independent segments, and therefore must spread independently. For this to be the case, they must lie on independent tiers. Before we demonstrate this, we shall briefly explain the way in which application of rules to Autosegmental representations is notated in Autosegmental Phonology.

The two basic representation-altering operations of Autosegmental Phonology are the addition of association lines to Autosegmental representations and the deletion of association lines from Autosegmental representations. Association lines which are added to a representation are drawn as dotted lines. Thus (16f) and (17c) denote representations to which a single association line has been added, and (16a–e), (17a,b) and (18) denote representations to which two association lines have been added. Where the addition of association lines to a representation incrementally 'links' a single item on one tier to successive objects on another tier, the single item is said to 'spread'. There are no instances of deletion in this example, so we shall not discuss it further.

We shall argue in detail that comparison with similar words of the same general phonological shape, such as 'tomb', 'root', 'loot' and so on, shows that the spread of backness, rounding and nasality is clearly phonologically distinctive, and cannot simply be attributed to automatic phonetic coarticulation effects. Consider the transcriptions in (15).

4.1 Rounding

Along with the proponents of Autosegmental Phonology, we regard it as uncontentious that there is a feature of liprounding (under whatever name) which is a primary articulation of vowels and a secondary articulation of consonants. A comparison of (15f) with (15a–e) shows that the spread of rounding from rounded vowels to neighbouring

a) room [ṽ̞ ŭ m̰]

b) loom [l̃ʲ̰ ũ̟ m̰ˠ]

c) zoom [z̰ ẽ̞ ũ m̰]

d) root [ṽ̞ e̞ u t̰]

e) loot [l̰ʲ e̞ ü̟ t̰ˠ]

f) lute [lʲ ë̞ ü̟ t̰ˠ]

g) rule [ṽ̞ ŭ lˠ]

h) feud [fʲ j̰ ë̞ ü̟ d̄ˀ]

i) cube [k̟ j̰ ë̞ ü̟ b̄ˀ]

j) rip [ṽ̞ ɪ pˠ]

k) red [ṽ̞ ɛ d̄ˀ]

Superscript ʲ and ˠ denote palatalised and velarised articulations respectively. Superscript ˀ denotes ingressive airstream (implosion), subscript ̰, ̞ and ̟ respectively denote voicelessness, advanced and retracted articulations.

Figure 15: Transcriptions of Guyanese English words.

consonants is not an automatic coarticulatory effect, but is a phonologically principled phenomenon.

Comparison of (15a), in which the coda is rounded, with (15g), (15h) and (15i), in which the codas are not rounded even though a rounded nucleus precedes, demonstrates that Coda rounding is not an automatic coarticulatory effect, but is a phonologically principled phenomenon. In accordance with Autosegmental Phonology's preference for autosegmental analyses of feature-spreading, the perseverative rounding of the coda in (15a) must be attributed to the spreading of the autosegmental feature [*rnd*].

There is no *phonetic* reason why rounding should not spread from the second vocalic element in (15f) to the first vocalic element and thence to the initial consonant. Consequently, something must block the spread of rounding to the onset of (15f). There is no reason to regard the onset itself as the locus of this blocking: (15b) and (15e) show that rounding is sometimes found with palatalised lateral onsets. Thus it must be the first vocalic element which blocks the forward spread of rounding in (15f). The only nonarbitrary way of blocking such a spread in within the terms of Autosegmental Phonology is to propose the presence of an adjacent autosegment which is associated to the skeletal tier in such a way that the N.C.C. would be violated if the spreading continued further. Thus it is not possible to derive (16g) from (16f). (This analysis also demonstrates that [*rnd*] is Autosegmental even if there are two V units in the Autosegmental representation of (15f).)

4.2 Nasality

A comparison of (15c) with (15a) and (15b) shows that nasality spreads from nasal coda consonants to vowels (an uncontentious analysis) and thence to 'liquid' sonorants /l/ and /r/. The absence of nasality in the onset of 'soom' (as well as 'soon', which behaves similarly) shows that this spreading is phonologically conditioned (17).

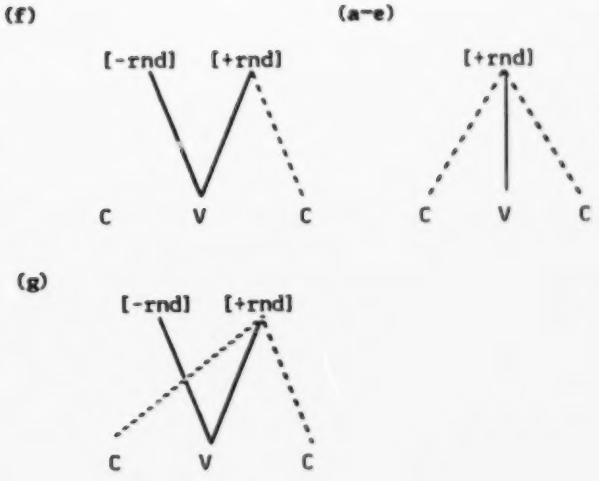


Figure 16: Rounding is autosegmental in Guyanese.

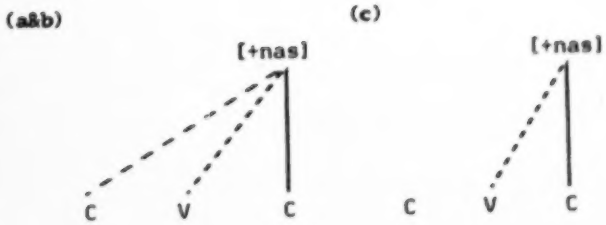


Figure 17: Nasality is autosegmental in Guyanese.

4.3 Backness

A comparison of (15a) with (15b) and (15d) with (15e) shows that the 'liquid' onsets /l/ and /r/ are systematically associated with the feature [±*back*], as is characteristic of many varieties of English (cf. Kelly and Local 1986, 1989: 74, 1989: 218-241). In this variety, /l/ is [−*back*] and /r/ is [+*back*]. Although the secondary articulation of onset consonants (notably obstruents) in English is attributable to the features of the vocalic nucleus, this is not the case with liquid onsets. The [−*back*] liquid remains [−*back*] before systemically [+*back*] vowels (15b) and the [+*back*] liquid remains [+*back*] before systemically [−*back*] vowels (15j,k). The distinctive association of liquid onsets with [±*back*] affects the nucleus and coda too, resulting in advanced vowel qualities and palatalised codas with the 'clear' liquid (15b), and retracted vowel qualities and velarised codas with the 'dark' liquid (15a). The spread of [±*back*] as far as the coda only applies in the case of coda consonants which are not lexically associated to [±*back*]. In the case of liquid codas, of course, spreading of [±*back*] from the nucleus is sometimes blocked (15g). Thus

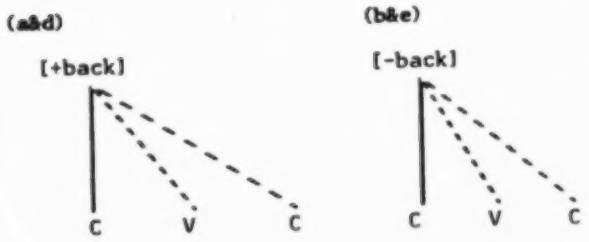


Figure 18: Backness is autosegmental in Guyanese.

[±*back*] is an autosegmental feature of liquids which in (15a) and (15b) spreads from the onset to the nucleus and thence to the coda (18). In the terms of Autosegmental Phonology, it is clear in the analysis of (15a) and (15b) that

- [±*rnd*], [±*back*] and [±*nas*] must be autosegments on separate tiers;
- liquid onsets are lexically associated with [±*back*], the nucleus with [±*rnd*], and the coda with [±*nas*]; and
- these three autosegments then spread to each of the other syllable terminals, as in (15a–e), (17a,b), and (18) to produce the Autosegmental representations portrayed in (14).

These interacting principles are each widely exemplified in several varieties of English, and although we present only a handful of critical examples here, many more may be found in Kelly and Local (1986, 1989).

Planar graphs are not in general adequate for Autosegmental representations of Guyanese English, because the Autosegmental representations of 'room' and 'loom' cannot be planar. Given that the principles which interact to produce this result are not particularly special and are individually attested elsewhere, we have no reason to believe that Guyanese English is either unnatural or special in this respect, and thus planarity (i.e. the No Crossing Constraint) is too severe to be a universal constraint on Autosegmental graphs.

Since Autosegmental Phonology is necessarily nonplanar, the No Crossing Constraint has no force, because all graphs, however complex, can be drawn in three dimensions without edges crossing. (The fact that some versions of Autosegmental Phonology employs 'paddle wheel' graphs, rather than unrestricted (i.e. Euclidean) graphs, does not affect this result.)

We conclude that the No Crossing Constraint is not a valid constraint at all, since it either incorrectly restricts the class of phonological graphs to planar graphs, or else it carries no force.

REFERENCES

Archangeli, D. 1985. Yokuts harmony: Evidence for coplanar representation in nonlinear phonology. *Linguistic Inquiry* 16. 335–372.

Chomsky, N. 1963. Formal properties of grammars. In R. D. Luce, R. R. Bush and E. Galanter (eds.) *Handbook of Mathematical Psychology* Volume II. New York: John Wiley.

Clements, G. N. 1985. The geometry of phonological features. *Phonology Yearbook* 2. 225–252.

Clements, G. N. 1986. Compensatory lengthening and consonant gemination in LuGanda. In Wetzels and Sezer, eds.

Clements, G. N. and S. J. Keyser 1983. *CV Phonology: a Generative Theory of the Syllable.* Linguistic Inquiry Monographs 9. Cambridge, Massachussetts: M.I.T. Press.

Culy, C. 1985. The complexity of the vocabulary of Bambara. *Linguistics and Philosophy* 8. 345–351.

Gibbons, A. 1985. *Algorithmic Graph Theory*. Cambridge: Cambridge University Press.

Goldsmith, J. A. 1976. *Autosegmental Phonology*. Bloomington: Indiana University Linguistics Club.

Goldsmith, J. A. 1985. Vowel harmony in Khalkha Mongolian, Yaka, Finnish and Hungarian. *Phonology Yearbook* 2. 253–275.

Halle, M. 1985. Speculations about the representation of words in memory. In V. A. Fromkin, ed. *Phonetic Linguistics: Essays in Honor of Peter Ladefoged*. New York: Academic Press.

Halle, M. and J.-R. Vergnaud 1980. Three dimensional phonology. *Journal of Linguistic Research* 1.1. 83–105.

Halle, M. and J.-R. Vergnaud 1987. Stress and the cycle. *Linguistic Inquiry* 18. 45–84.

Hammond, M. 1988. On deriving the well-formedness condition. *Linguistic Inquiry* 19. 319–325.

Hayes, B. 1989. Compensatory lengthening in moraic phonology. *Linguistic Inquiry* 20. 253–306.

Huybregts, R. 1984. The weak inadequacy of context-free phrase structure grammars. In G. de Haan, M. Trommelen, and W. Zonneveld, eds. *Van Periferie naar Kern*. Dordrecht: Foris.

Kaye, J. D. 1985. On the syllable structure of certain West African languages. In D. L. Goyvaerts, ed. *African Linguistics: Essays in Memory of M. W. K. Semikenke*. Amsterdam: John Benjamins.

Kelly, J. and J. K. Local. 1986. Long-domain resonance patterns in English. In *International Conference on Speech Input/Output; Techniques and Applications* IEE Conference Publication No. 258.

Kelly, J. and J. K. Local. 1989. *Doing Phonology*. Manchester: Manchester University Press.

Kuratowski, G. 1930. Sur le problème des courbes gauches en topologie. *Fundamenta Mathematicæ* 15. 271–83.

Lowenstamm, J. and J. Kaye. 1986. Compensatory lengthening in Tiberian Hebrew. In Wetzels and Sezer, eds.

Manaster-Ramer, A. 1983. The soft formal underbelly of theoretical syntax. *C.L.S.* 19. 256–263.

McCarthy, J. 1979/1982. *Formal Problems in Semitic Phonology and Morphology*. Bloomington: Indiana University Linguistics Club.

McCarthy, J. 1981. A prosodic theory of nonconcatenative morphology. *Linguistic Inquiry* 12. 373–418.

McCarthy, J. 1986. OCP effects: Gemination and antigemination. *Linguistic Inquiry* 17. 207–263.

McCarthy, J. 1989. Linear order in phonological representation. *Linguistic Inquiry* 20. 71–99.

Prince, A. S. 1984. Phonology with tiers. In M. Aronoff and R. Oehrle, eds. *Language Sound Structure*. Cambridge, Massachusetts: M.I.T. Press.

Prince, A. S. 1987. Planes and copying. *Linguistic Inquiry* 19. 491–509.

Pulleyblank, D. 1986. *Tone in Lexical Phonology*. Dordrecht: D. Reidel.

Pulleyblank, D. 1988. Vocalic underspecification in Yoruba. *Linguistic Inquiry* 19. 233–270.

Pullum, G. and Gazdar, G. 1982. Natural languages and context free languages. *Linguistics and Philosophy* 4. 471–504.

Rosen, B. K. 1977. Arcs in graphs are NOT pairs of nodes. *SIGACT News*. Fall 1977. 25–27.

Sagey, E. 1988. On the ill-formedness of crossing association lines. *Linguistic Inquiry* 19. 109–118.

Shieber, S. 1985. Evidence against the context-freeness of natural languages. *Linguistics and Philosophy* 8. 333–343.

van der Hulst, H. and N. Smith 1982. Introduction to H. van der Hulst and N. Smith, eds. *The Structure of Phonological Representations vol. I*. Dordrecht: Foris.

Wetzels, L. 1986. Phonological timing in Ancient Greek. In Wetzels and Sezer, eds.

Wetzels, L., and E. Sezer, eds. 1986. *Studies in Compensatory Lengthening*. Dordrecht: Foris.

Wilson, R. J. 1985. *Introduction to Graph Theory*. London: Longman.